UG–C-2287 **BMS–31X/ BMC–31X**

U.G. DEGREE EXAMINATION — DECEMBER, 2023.

Mathematics

Third Year

REAL AND COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — $(3 \times 3 = 9$ marks)

Answer any THREE questions out of five questions in 100 words.

All questions carry equal marks.

1. What is open ball?

2. Define uniformly continuous.

3. What is meant by Riemann integral?

4. Write a short note on conformal mapping.

5. Define residues.

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Show that every convergent sequence is a cauchy sequence in any metric space.

7. Show that every constant function is constant.

8. State and prove Rolle's theorem.

9. Find the fixed point of $W = \frac{2z+i}{z-(1+i)}$

10. Evaluate for $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\, d\theta,\ (a>b>0)$

PART C — ($4 \times 10 = 40$ marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. State and prove Holder's inequality.

12. Show that any compact subset of a metric space is closed and bounded.

13. State and prove the chain rule for differentiable function.

14. If f is analysitc in a region Ω. Then verify the real and imaginary parts satisfy the C-R equations.

15. State and prove cauchy's integral formula.

16. Show that any complete metric space is of the second category.

17. State and prove the second fundamental theorem of integral calculus.

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U.G. DEGREE EXAMINATION – DECEMBER 2023

Mathematics

Third Year

LINEAR ALGEBRA AND BOOLEAN ALGEBRA

Time : 3 hours Maximum marks : 70

SECTION A — (3 × 3 = 9 marks)

Answer any THREE questions out of Five questions in 100 words

All questions carry equal marks.

1. Define vector space.
2. What is meant by linear span?
3. Write a short note on orthogonal.
4. Define symmetric bilinear form.
5. Write a note on boolean ring.

SECTION B — (3 × 7 = 21 marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. Let V be a vector space over a field F. Then show that a nonempty subset W of V is a subspace of V. Iff $u, v, \in w$ and $\alpha, \beta \in F \Rightarrow \alpha u + \beta v \in W$.

7. Let $T : V \to W$ be a linear transformation, then prove that $\dim V = \text{rank}\, T + \text{nulity}\, T$.

8. The norm defined in an inner product space V, then prove that $\|\langle x, y\rangle\| = \|x\|\|y\|$.

9. Reduce the quadratic form $x_1^2 + 4x_1 x_2 + 4x_1 x_3 + 4x_2^2 + 16 x_2 x_3 + 4x_2^3$ to the diagonal form.

10. Let L be a lattice and let $a, b, c, d, \in L$. Then show that $a \le b$ and $c \le d \Rightarrow$ (a) $a \vee c \le, b \vee d$ and (b) $a \wedge c \le b \wedge d$.

SECTION C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. State and prove the fundamental theorem of homomorphism.

12. Let V be a vector space over F and W a subspace of V. Let $\frac{v}{w} = \{W + v / v \in V\}$, then show that $\frac{v}{w}$ is a vector space over F under the following operation.

    (a) $(W + v_1) + (W + v_2) = (W + v_1 + v_2)$

    (b) $\alpha\ (W + v_1) = (W + \alpha v_1)$

13. Let V be a finite dimensional vector space over a field F. Let W be a subspace of V then prove that

    (a) $\dim W \leq \dim V$

    (b) $\dim \frac{V}{W} = \dim V - \dim W$.

14. Let V be vector space over a field F. Let $S = \{v_1, v_2, \ldots. v_n\} \subseteq V$. Then show that the following are equivalent

    (a) S is a basis f or V

    (b) S is a maximal linear by independent set

    (c) S is a minimal generating set.

15. Let W_1 and W_2 be a subspaces of a finite dimensional inner product space. Then prove that the following

(a) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$

(b) $(W_1 \cap W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$

16. Let V be a vector space over a field F then show that $L\ (V,V,F)$ is a vector space over F under addition and scalar multiplication defined by $(f+g)(u,v) = f(u,v) + g(u,v)$ and $(\alpha f)(u,v) = \alpha f(u,v)$.

17. Show that the lattice of normal subgroup of any group is a modular lattice.

UG–C-2291	**BMS–35X/ BMC –34X**

U.G. DEGREE EXAMINATION — DECEMBER, 2023.

Mathematics

Third Year

GRAPH THEORY

Time : 3 hours   Maximum marks : 70

PART A — ($3 \times 3 = 9$ marks)

Answer any THREE questions out of Five questions in 100 words.

All questions carry equal marks.

1. Define isomorphism of a graph.

2. What is meant by bipartite graph?

3. What is closure of a graph?

4. Write a note on chromatic polynomial of G.

5. Define euler diagraph.

PART B — ($3 \times 7 = 21$ marks)

Answer any THREE questions out of Five questions in 200 words.

All questions carry equal marks.

6. If G is simple graph and $\delta \geq (p-1)/2$, then prove that G is connected.

7. Show that an edge of G is a cut edge of G if and only if e is contained in no cycle of G.

8. In any graph with $\delta \geq 0$, then prove that $\alpha' + \beta' = P$.

9. If G is connected plane graph, then show that $p - q + r = 2$.

10. In a diagraph D, show that sum of the indegrees of all the vertices is equal to the sum of their out degrees, each sum being equal to the number of arcs in D.

PART C — (4 × 10 = 40 marks)

Answer any FOUR questions out of Seven questions in 500 words.

All questions carry equal marks.

11. Show that the maximum number of edges among all p vertex simple graphs with no triangle is $[p^2/4]$.

12. Discuss briefly a connected graph G with atleast two vertices is a tree if and only if its degree sequence (d_1, d_2, ..., d_p) satisfies the condition $\sum_{i=1}^{p} d_i = 2(p-1)$ with $d_i > 0$ for each i.

13. If G is a graph with $p \geq 3$ and $\delta \geq p/2$, then prove that G is hamiltonian.

14. Show that every planar graph is 5-vertex colourable.

15. Discuss briefly every tournament has a directed hamilton path.

16. Show that a graph is bipartite if and only if it contains no odd cycle.

17. State and prove the pigeonhole principle theorem.

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